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What Is a Liquidity Crisis?*

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This paper investigates the role of liquidity. It is argued that the firm holds money as a guarantee of solvency to its contractual partners. By holding liquid assets the firm is able to increase the efficiency of its contracts. Variations in the opportunity cost of holding money cause variations in the firm's demand for money and consequent variations in the frequency of contract failures. This mechanism is suggested as an alternative explanation of the observed relationship between money, income, and interest rates. *Journal of Economic Literature* Classification Numbers: 023, 026, 131, 311, 315.

1. INTRODUCTION

One of the more striking relationships that one observes in macroeconomic time series is the existence of a strong negative correlation between measures of economic activity and lagged interest rates. For postwar U.S. time series, this relationship has been documented by Sims [18] and by Litterman and Weiss [14], both of whom find that, in vector autoregressions on small sets of aggregate data, high interest rates precede a recession by six to nine months.

The results of Litterman and Weiss are particularly interesting to a theorist because they indicate that there exists a relationship between the *nominal* rate of interest and a measure of aggregate economic activity after correcting for anticipated inflation. This paper offers an explanation of this relationship by explicitly modelling the role of liquidity in production.

In earlier work [5, 6], I have argued that high *real* interest rates may permanently lower equilibrium rates of resource utilization by increasing

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the probability that any single firm will become bankrupt. In the current paper, I extend this idea to analyze the effect of *liquidity* as opposed to *bankruptcy*, and I present a theory of contracts under asymmetric information which suggests that a firm will be more likely to lay off a worker the less liquid are its assets. This theory suggests that, when nominal interest rates are high, firms and workers will *choose* to write contracts which are more likely to result in underemployment.

One may envisage a range of economic environments that distinguish between "liquid" and "illiquid" assets, for example, spatial separation of the type discussed by Townsend [20], cash-in-advance constraints of the type analyzed in Lucas [15], or legal restrictions theory as discussed by Bryant and Wallace [2] and Wallace [21]. In a companion paper [7], I analyze an explicit general equilibrium model which uses legal restrictions theory to investigate the impact of monetary policy. This companion paper contains a more careful analysis of the general equilibrium structure of a related model but a more rudimentary analysis of contracts. The current paper is set in partial equilibrium, but the reader is referred to [7] for a discussion of the implications of this analysis for natural rate theory and the impact of monetary policy on equilibrium rates of resource utilization.[1]

The paper is closely related to earlier work in asymmetric information contracts by Grossman and Hart [9], Azariadis [1], Chari [3], and Grossman, Hart, and Maskin [10]. The techniques employed to analyze an optimal contract in continuous state space are developed in Green and Kahn [8] and related work that investigates the properties of contracts with limited liability can be found in Sappington [17], Kahn and Scheinkman [12], Leach [13], and Farmer [5, 6]. The point of departure from this literature is the distinction that I make between those incentive conflicts that arise because the wealth of an agent may be limited and those conflicts that arise because the wealth of an agent may be private information. I shall reserve the term bankruptcy to refer to that class of incentive conflicts that arise from limited wealth (these are the issues discussed in the literature cited above), and I shall reserve the term liquidity crisis to refer to the class of incentive conflicts that arise from limited observability of wealth.

In formal terms the distinction between bankruptcy and liquidity amounts to allowing a firm to post a bond as a means of guaranteeing its good behavior. The assumption that bankruptcy is not a binding constraint means that the wealth of the firm is sufficiently large for there to be no limit on the potential size of this bond. The assumption that liquidity *may* be a problem means that bonding is costly. The formal analysis of costly

[1] Both of these papers draw heavily on a joint project with Costas Azariadis in which we analyze the properties of a pure exchange version of this model.

bonding may be of independent interest to the reader with an interest in the principal/agent literature and its applications to other areas of economics.

2. The Assumptions

I shall assume that two parties plan to engage in a joint productive venture that will yield a return $y(l, s)$ one period hence. The term s represents a stochastic productivity shock with density function $h(s)$ and support in the interval $[\underline{s}, \bar{s}]$. The term l represents the quantity of an input into the production process that will be supplied by one party to the contract. For the sake of concreteness, I shall sometimes refer to l as labor services, to the supplier of l as the worker, and to the second party to the contract as the firm/entrepreneur, but the analysis will apply equally well to a contractual relationship between a firm and its suppliers of raw materials. Specifically, $y(l, s)$ takes the form

$$y(l, s) = sf(l), \tag{1}$$

where $f' > 0$, $f'' < 0$, and f is C^2.

Assume that the worker and the firm must contract in advance for the supply of l and that, ex post, the contract will be enforceable. Assume also that once uncertainty is resolved there is no opportunity for either party to relocate with an alternative partner, i.e., the relationship that will be entered into by the firm and its supplier will generate pair-specific rents.

A contract $\delta(s)$ is a compensation package $\omega(s)$ and an employment level $l(s)$ as functions of the state of nature,

$$\delta(s) \equiv \{\omega(s), l(s)\}, \tag{2}$$

and the firm's contracting problem is to design a contract $\delta(s)$ which allocates these pair-specific rents across different states of nature.

Much of the incentive-conflict literature focuses on the interaction between efficient risk sharing and the allocation of correct incentives for informed agents to supply effort. However, this is not the only source of potential incentive conflict and, in this paper, risk sharing does not play an important role.[2] For this reason, and in order to make the analysis as clear

[2] See, for example, Sappington [17], who analyzes a principle agent relationship in which limited wealth, rather than risk aversion, is the primary source of incentive conflict.

as possible, I shall assume that both the entrepreneur and the worker are risk neutral and that the worker has the simple indirect utility function[3]

$$V = \omega(s) - V[l(s)], \tag{3}$$

where $V' > 0$, $V'' > 0$, and V is C^2.

Similarly, the firm's objective function is equal to its expected net present value and the firm's problem is to design a contract that maximizes this quantity subject to certain feasibility constraints that are outlined below.

(i) *Competition*

To close the model, I shall divide the rents to the relationship arbitrarily between the worker and the entrepreneur by assuming that the worker has an alternative opportunity that is worth $\bar{V}$ in utility units. This is a partial equilibrium assumption that would be replaced in general equilibrium by an assumption about entry into alternative activities. Its effect is to require that a feasible contract should offer the worker at least $\bar{V}$ ex ante, i.e.,

$$E_s[\omega(s) - V[l(s)]] \geqslant \bar{V}. \tag{4}$$

(ii) *No Bankruptcy*

To distinguish the idea of "liquidity" from the possibility that the firm may, in some states of nature, become bankrupt I make the, assumption

$$\bar{V} \leqslant A. \tag{5}$$

Inequality (5) implies that the entrepreneur is sufficiently wealthy to be able to guarantee the outcome of productive uncertainty with its worker across all states of nature.

(iii) *Liquidity Constraint*

To capture the idea that some assets are more liquid than others, I shall assume that the value of A is private information that will not become common knowledge until some date in the future, after the completion of the contract between the worker and the firm and after the realization of the productive uncertainty s. This assumption implies that the firm will be unable to issue paper liabilities backed by its assets A. Although A cannot be observed by the worker, I shall assume that it *can* be observed, at some cost, by a third party called a bank. The bank monitors the behavior of the

[3] It is well known (Cooper [4]) that over- or underemployment may result from labor contracts under asymmetric information as leisure is a normal or inferior good. However, the introduction of a risk neutral third party removes this implication, hence, this utility function may be less restrictive than it at first appears.

firm and it guarantees that the firm is solvent by issuing a loan. In effect the bank substitutes its own paper liabilities, which are acceptable to third parties in exchange, for the liabilities of the firm that are not.[4] These assumptions are captured by the inequalities

$$\omega(s) \leqslant m + y(l, s), \tag{6}$$

where m represents that part of the entrepreneur's assets that has been monetized by obtaining a loan from the bank.

(iv) *Incentive Compatibility*

The incentive conflict that I propose to explore arises from the interaction of the liquidity constraint with the assumption that productive uncertainty is private information that is observed only by the entrepreneur. Assume that s is observed, ex post, by the firm and let s^* be the state that is announced by the entrepreneur to the worker. From the revelation principle (see Myerson [16] or Harris and Townsend [11]), it follows that one may restrict attention to the set of contracts for which

$$\operatorname*{Max}_{s^*} \; y(l(s^*), s) - \omega(s^*) \tag{7}$$

s.t.

$$l \geqslant 0 \tag{8}$$

occurs at $s = s^*$ for all s.

An incentive compatible contract must therefore satisfy the conditions[5]

$$(sf_l l_s - \omega_s) l = 0, \qquad l \geqslant 0 \tag{9}$$

$$l_s \geqslant 0. \tag{10}$$

Equation (9) follows from the first-order conditions to (7) and (10) comes from totally differentiating (9) and comparing the result with the second-order conditions for a maximum. Conditions (9) and (10) are both necessary and sufficient for a contract $\{l, \omega\}$ to represent a solution to (7).

[4] I am not explaining *why* the liabilities of the bank are more acceptable although this is clearly a central issue. One possible assumption is that bankers are "trustworthy" individuals who are nominated by the community to perform a monitoring role, i.e., one might base a more fundamental theory on the idea of reputation. I regard questions of this kind as important, but I am not yet able to address them in a satisfactory manner.

[5] This is, essentially, the technique employed in Green and Kahn [8]. It assumes continuity of $\{\omega(s), n(s)\}$ which, strictly speaking, cannot be guaranteed a priori. However, in this problem one may show that there is no discontinuous contract that dominates. I am grateful to John Moore for drawing my attention to this issue.

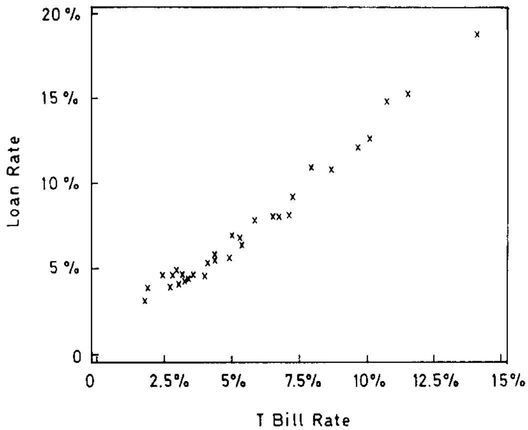

FIG. 1. A scatter diagram showing the relationship between the interest rate on T bills and the interest rate on prime business loans.

(v) *The Cost of Credit*

I assume that the borrowing of the firm puts an upper limit on payments to the worker and that the cost of a loan is equal to the loan rate of interest r^{L}. Most firms hold liquid assets and at the same time they borrow from banks.[6] The cost of borrowing is related to the difference between the loan rate of interest r^{L} and the deposit rate of interest r^{D}, which move closely with each other in a linear relationship. This relationship is captured by the assumption

$$r^{\mathrm{D}} = (1-\theta) r^{\mathrm{L}} \tag{11}$$

where

$$0 < \theta < 1. \tag{12}$$

Since this assumption plays an important part in the analysis I have included, in Fig. 1, a graph of the relationship between the T bill rate and the rate of interest on prime business loans for post war U.S. data. A regression of prime rate on the T bill rate yields a regression coefficient of 1.25 with a *t* statistic of 32. There are a number of possible theoretical reasons why one would expect the loan rate to be a simple multiple of the

[6] In 1981, for example, businesses held \$135 billion in the form of T bills and bank accounts and took out commercial bank loans of \$327 billion ("Economic Report of the President").

interest rate on liquid assets, one of which is explored in [7]. For the purposes of this analysis, however, it is sufficient to note that such a relationship exists and that it implies that the spread between borrowing and lending rates is high when interest rates are high.[7]

3. The Economic Environment

I shall assume that the firm and its worker meet in a competitive environment in which the worker faces an alternative opportunity that is worth $\bar{V}$. The entrepreneur must offer a contract $\delta(s)$ that pays at least $\bar{V}$ in order to entice the worker to forego this opportunity. At the same time that the firm enters into a contract with its worker/supplier, it negotiates a loan from a bank at interest rate r^{L}. The loan from the bank places a lower bound on the payments that may be made to the worker, and its reduces the portion of compensation payments that are state dependent. Since state-dependent payments may introduce inefficient resource allocations in problems that involve asymmetric information, it turns out that a loan may have some value to the firm even though it is costly to obtain.

After negotiating a loan from the bank and writing a contract with the firm, the worker is assumed to lose his or her outside opportunity. At this point, uncertainty is revealed to the firm and the terms of the contract are fulfilled.

The optimal contract is a compensation schedule $\hat{\omega}(s)$ and a supply of labor schedule $\hat{l}(s)$ that maximizes the expected utility of the entrepreneur subject to constraints (i)–(v) outlined above, that is, an optimal contract may be represented as a solution to the following programming problem:

Problem P. Let $X \equiv \{l(s), \omega(s)\}$ be a vector of state variables, $b \equiv \{l_0, l_1, \omega_0, \omega_1, m\}$ be a vector of control parameters, and let u be a control variable. Then an optimal contract is a solution to

$$\operatorname*{Max}_{u,b} \int_s [sf(l) - \omega + m]\, h\, ds(1 + r^{\mathrm{D}}) - (1 + r^{\mathrm{L}})m \tag{13}$$

[7] The most obvious explanation of this relationship is the presence of reserve requirements which act as a proportional tax on the banking industry. Note that, although these data refer to the T bill rate rather than a deposit rate, the T bill rate moves very closely with money market deposit rates and T bills are very close substitutes for deposits in the portfolios of firms.

$$\int_s (\omega - V(l))\, h\, ds \geqslant \bar{V} \tag{14}$$

$$A \geqslant \bar{V} \tag{15}$$

$$A \geqslant m \tag{16}$$

$$\frac{d\omega}{ds} = sf_l u \tag{17}$$

$$\frac{dl}{ds} = u \tag{18}$$

$$\begin{aligned} &\text{(i)} && \omega(\underline{s}) = \omega_0 \\ &\text{(ii)} && \omega(\bar{s}) = \omega_1 \\ &\text{(iii)} && l(\underline{s}) = l_0 \\ &\text{(iv)} && l(\bar{s}) = l_1 \end{aligned} \tag{19}$$

$$m + \underline{s}f(l_0) - \omega_0 \geqslant 0 \tag{20}$$

$$l_0 \geqslant 0 \tag{21}$$

$$u \geqslant 0. \tag{22}$$

Expression (13) represents the objective function of the firm. The term in square brackets represents the real value of the firm's liquid assets on which it receives interest at the deposit rate r^{D}. The second term in expression (13) represents the cost of obtaining a loan. The real value of non-marketable assets A has been suppressed from this expression since its only purpose is to place a bound on the amount of borrowing that the firm may undertake (inequality (16)), which will never be binding in light of the no bankruptcy assumption (inequality (15)).

I have directly imposed first-order incentive compatibility in Eqs. (17) and (18). The non-negativity constraint $l(s) \geqslant 0$ cannot be directly imposed since the control variable does not enter the constraint. It can, however, be imposed indirectly by constraining $l(\underline{s})$ to be non-negative and imposing the second-order incentive compatibility constraint $dl/ds \equiv u \geqslant 0$. (See Takayama [19, p. 666, footnote 16] for a discussion of this point.) Similarly the liquidity constraint (Eq. (20)) should hold at all values of s. Notice, however, that if the constraint holds at $\underline{s}$ then it will hold for all $s > \underline{s}$ since profits are an increasing function of s if the contract is incentive compatible, i.e.,

$$\frac{d}{ds}[sf - \omega] = f \geqslant 0 \tag{23}$$

since

$$sf_l l_s - \omega_s = 0. \tag{24}$$

In the absence of the liquidity and incentive compatibility constraints, this problem has a solution which will be referred to as a first best optimum (FBO).

DEFINITION 1. A FBO contract $\{l^*, \omega^*\}$ is characterized by the conditions

$$(sf_l(l^*) - V_l(l^*))\, l^* = 0 \tag{25}$$

$$l^* \geqslant 0 \tag{26}$$

$$E_s[\omega^*] = E_s[V(l^*)] + \bar{V}. \tag{27}$$

Equations (25) and (26) are the Kuhn–Tucker conditions for efficient production. Employment is found by equating the marginal product to the marginal disutility of work provided this yields on interior solution. Equation (27) permits any wage contract for which the expected utility in employment is equal to the worker's next best alternative. There will thus be a continuum of admissible contracts. This result reflects the assumption of risk neutrality which causes the worker to be indifferent about the distribution of payments across states of nature.

4. THE SOLUTION TO THE FIRM'S PROBLEM

It is well known that, even under conditions of asymmetric information, contracts between risk neutral agents can be made first best. The solution is for the firm to guarantee a constant utility level in all states by designing a compensation schedule which has the same slope as the disutility of effort schedule $V(\cdot)$. Ex post, the firm would equate the marginal increment in the compensation schedule to the marginal product and ex post optimizing behaviour would cause it to choose an efficient employment level.

However, this scheme will not be optimal if the firm faces a liquidity constraint. The worker cares about the slope of the payment schedule only in as much as it affects the total expected utility level which he or she would obtain by taking a job at the firm. If the slope of the payment schedule is determined by the requirement that it should generate efficient production then the absolute level of the schedule must be determined by the condition that the worker's expected utility is equal to his/her next best opportunity. But in the worst state the worker's compensation is bounded by the liquidity constraint and it is costly to hold money because the firm must borrow at the loan rate of interest. By paying the worker a little more than his or her marginal product in good states of nature the firm is able to relax its liquidity requirements in low states. But this increment causes the slope of the compensation schedule to exceed the disutility of work and it drives a wedge between the marginal rates of transformation in production and consumption; the value-maximizing firm will trade off the benefit to liquidity in terms of productive efficiency against the cost in terms of the opportunity cost of holding money.

This argument is formalized in Theorem 1. Before stating this theorem, it is helpful to impose a regularity condition on the density function of s which helps to guarantee an interior solution:

$$z_s \geqslant 0, \tag{R1}$$

where $z(s) \equiv h(s)/(1 - H(s))$ is the hazard function.

Equation (R1) is satisfied by a large class of density functions and, whilst it is not required in order to generate the results, it does help one to cut down on notation.[8] It is also useful to define the following concepts.

DEFINITION 2.

$$i \equiv \frac{r^{\mathrm{L}} - r^{\mathrm{D}}}{1 + r^{\mathrm{L}}}. \tag{28}$$

The variable i represents the opportunity cost of liquidity. Notice that by assumptions (11) and (12) it follows that

$$i = \frac{\theta r^{\mathrm{D}}}{(1 - \theta + r^{\mathrm{D}})}, \tag{29}$$

where $\delta i/\delta r^{\mathrm{D}} > 0$.

DEFINITION 3.

$$\psi(i, s) = s - \frac{i}{z(s)}. \tag{30}$$

[8] Equation (R1) is sufficient (but not necessary) to rule out pooling equilibria in which a number of states may be associated with the same employment level. In the absence of (R1) the second-order incentive compatibility constraint $l_s \geqslant 0$ may become binding in some states. Equation (R1) is satisfied by a large class of density functions including the Exponential, the Gamma and Weibull with degrees of freedom parameter larger than 1, the Normal Distribution, La Place, and the Uniform.

THEOREM 1. *Assume* (R1) *and* $i > 0$. *Then the solution to P is given by*

(i) $$\hat{l}(s) = l^*(\psi),$$

(ii) $$\hat{\omega}_0 = m + \underline{s}f(\hat{l}_0),$$

(iii) $$\hat{\omega}(s) = \omega_0 + V(\hat{l}(s)) + g(i, s),$$

where $g(i, s) \equiv \int^s (i/z(x)) f_l(\hat{l}(x)) \hat{l}_x \, dx$ *and where* m *is found from the constraint* $E_s[\hat{\omega}(s) - V(\hat{l}(s))] = \bar{V}$.

The proof is contained in the Appendix.

5. DISCUSSION OF THE RESULTS

Part (i) of the theorem defines the employment rule in the optimal contract. $l^*(s)$ is the amount of labor that would be hired in state s if the contract were FBO. The constraints which were described above cause the firm to act as if the state of nature were actually worse than this, i.e., the firm will hire the same number of labor hours which would be employed in an efficient contract in which the state was ψ rather than s. Since ψ is less than s in all states except for the highest state ($s = \bar{s}$), the firm will employ less labor hours than would be observed in an efficient contract.

Departures from (first best) optimality are related to the interest rate, to the probability density function, and to one minus the cumulative density function. One may use the difference—between the state which actually occurs and the state which would have to occur in order for the actual employment level to be first best—as a measure of the departure of the contract from first best optimality. This quantity is given by

$$s - \psi = \frac{i(1 - H(s))}{h(s)}. \tag{31}$$

In states in which this difference is large, the payment function is steep and the employment level is particularly inefficient. The worker is only concerned about the *level* of compensation which he or she receives but departures from efficient employment occur when the *slope* of this function deviates from the first best contract. A second best contract will, therefore, generate the greatest departures from first best employment in those states for which a variation in the slope will have the greatest impact on the level. This effect is captured by the term $1 - H(s)$, which tends to generate large increments in compensation in low states. Departures from efficient employment are particularly valuable in low states because they increment the *level* of the compensation schedule *at all higher states*. In states which are very likely, however, departures from the FBO are very costly because they generate large expected losses. This second effect is captured by the term $h(s)$ which reduces the departure from efficient employment in those states which have a high probability of occurring.

Departures from efficiency are only generated if it is costly to hold money. If the interest rate is zero, then the firm will pursue an efficient employment policy ($s = \psi$) and it will hold enough liquid assets (at zero cost) to guarantee a constant utility level to the worker in all states. As the interest rate increases, however, the firm economizes on cash balances, which has the effect of shifting down the compensation schedule in all states. In order to compensate the worker for this shift, the schedule becomes steeper so that the worker will receive a larger share of revenues in high states. But this tilt in the compensation schedule is achieved at the cost of some productive inefficiency as the increment in the slope of the schedule drives a wedge between the marginal rates of transformation in production and in consumption.

6. Conclusion

I began this paper by describing the relationship that exists between the nominal interest rate and the level of economic activity. Macroeconomists usually explain this relationship with theories which rely on price stickiness that is very difficult to reconcile with rational choice. The ideas that I have introduced above represent an attempt to persuade the reader that we might more profitably explain the interest rate–output relationship by modelling the role of money in the productive process, that is, we should try to explain how the interest rate affects aggregate supply.

Appendix: Proof of Theorem 1

(i) *The Employment Rule*

By Hestenes' theorem (Takayama [19, p. 658]) if $[\hat{b}, \hat{u}(s), \hat{x}(s)]$ is a solution to P, then there exist multipliers $\lambda, p_1(s), p_2(s), q_1(s), q_2(s), q_3(s)$ not vanishing simultaneously and functions L and ψ where

$$L \equiv H + q_1[m + \underline{s}f(l_0 - \omega_0)] + q_2 l_0 + q_3 u \tag{A1}$$

with H defined as

$$H \equiv [(1 + r^{\mathrm{D}})(sf(l) - \omega) + \lambda(\omega - V(l))]\, h + p_1 s f_l u + p_2 u \tag{A2}$$

and

$$\psi(b) \equiv m(r^{\mathrm{D}} - r^{\mathrm{L}}). \tag{A3}$$

Note that a multiplier p_0 should be associated with the term $(sf(l) - \omega)$ in (A2). I implicitly assume that a normality condition holds (see Takayama [19, pp. 612–613]) which allows one to set $p_0 = 0$. First-order conditions for the maximization of L w.r.t. u imply

$$q_3 + p_1 sf_l + p_2 = 0. \tag{A4}$$

The Euler/Lagrange equations for the co-state variables are given by

$$\frac{dp_1}{ds} = ((1 + r^{\mathrm{D}}) - \lambda)\, h \tag{A5}$$

$$\frac{dp_2}{ds} = (-sf_l(1 + r^{\mathrm{D}}) + \lambda V_l)\, h - p_1 sf_{ll} u. \tag{A6}$$

I assume $l_s \equiv u > 0$ and derive the optimal employment rule which is implied by this assumption. I then show that if (R1) holds (an increasing hazard rate) this assumption is valid for all values of $l(s) > 0$. This amounts to demonstrating that (R1) is sufficient to rule out pooling equilibria in this problem, since $u > 0$ in the optimal contract implies that a different employment level will be associated with every state. If $\hat{u} > 0$ then $q_3 = 0$. One may then totally differentiate (A4), using (A5) and (A6) to derive

$$sf_l - V_l = \frac{p_1 f_l}{h\lambda}. \tag{A7}$$

The next step is to obtain an expression for $p_1 f_l / h\lambda$ by integrating the Euler equation (A5)

$$p_1(s) = p_1(\underline{s}) + ((1 + r^{\mathrm{D}}) - \lambda)\, H(s). \tag{A8}$$

The transversality conditions for the problem are given by

$$\frac{-\partial \Psi}{\partial b_j} + \left[\sum_{i=1}^{2} p_i(s) \frac{\partial x_i(s)}{\partial b_j} \right] \Bigg|_{s=\underline{s}}^{s=\bar{s}} = \int_{\underline{s}}^{\bar{s}} \frac{\partial \hat{L}}{\partial b_j}\, ds, \tag{A9}$$

where b_j is the jth element of $b \equiv [l_0, l_1, \omega_0, \omega_1, m]$. The transversality conditions for m imply

$$(r^{\mathrm{L}} - r^{\mathrm{D}}) = \int_{\underline{s}}^{\bar{s}} q_1\, ds \equiv Q_1 \geqslant 0, \tag{A10}$$

where $Q_1 \equiv {}_{\underline{s}}^{\bar{s}}\int q_1 \, ds \geqslant 0$ and the inequality follows from the Kuhn–Tucker conditions. Similarly the transversality condition for ω_0 is given by

$$p_1(\underline{s}) = \int_{\underline{s}}^{\bar{s}} q_1 \, ds \equiv Q_1. \tag{A11}$$

Combining (A10) and (A11) we may establish that

$$p_1(\underline{s}) = (r^{\mathrm{L}} - r^{\mathrm{D}}). \tag{A12}$$

Finally, the transversality condition for ω_1 implies $p_1(\bar{s}) = 0$. Hence from (A8) evaluated at $\bar{s}$ it follows that

$$p_1(\underline{s}) = (\lambda - (1 + r^{\mathrm{D}})). \tag{A13}$$

Rewriting (A8) using (A13) and (A12) yields

$$p_1(s) = (1 - H(s))(r^{\mathrm{L}} - r^{\mathrm{D}}). \tag{A14}$$

Returning now to (A7) and noting that by definition $i \equiv (r^{\mathrm{L}} - r^{\mathrm{D}})/(1 + r^{\mathrm{L}})$ and $z(s) \equiv h(s)/(1 - H(s))$ one finds (using (A14))

$$\frac{p_1}{h\lambda} = \frac{i}{z(s)} \tag{A15}$$

hence from (A7)

$$\left[s - \frac{i}{z(s)} \right] f_l = V_l. \tag{A16}$$

It remains to demonstrate that $l_s \equiv u > 0$ for the employment rule implied by (A16) and thus the premise $q_3 = 0$ is valid. But by totally differentiating (A16) w.r.t. s it may be shown that $z_s \geqslant 0$ is sufficient to imply $l_s > 0$ along the optimal contract. But note that (A16) is the FBO contract with s replaced by $\psi \equiv s - i/z(s)$—this establishes part (i) of the theorem.

(ii) *and* (iii) *The Compensation Rule*

Part (ii) follows from (A10) and (A11) which imply $q_1 > 0$ if $i > 0$ and therefore the liquidity constraint (20) will hold with equality.

Part (iii) follows from substituting Eqs. (18) and (A16) into the Euler equation (17) and integrating the resulting expression. This establishes Theorem 1.

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